

Connectivity statistics of store-and-forward inter-vehicle communication

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Abstract—Inter-vehicle communication (IVC) enables vehicles to exchange messages within a limited broadcast range and thus self-organize into dynamical vehicular ad hoc networks. For the foreseeable future, however, a direct connectivity between equipped vehicles in one direction is rarely possible. We therefore investigate an alternative mode in which messages are stored by relay vehicles traveling in the opposite direction, and forwarded to vehicles in the original direction at a later time. The wireless communication consists of two “transversal” message hops across driving directions. Since direct connectivity for transversal hops and a successful message transmission to vehicles in the destination region is only a matter of time, the quality of this IVC strategy can be described in terms of the distribution function for the total transmission time. Assuming a Poissonian distance distribution between equipped vehicles, we derive analytical probability distributions for message transmission times and related propagation speeds for a deterministic and a stochastic model of the maximum range of direct communication. By means of integrated microscopic simulations of communication and bi-directional traffic flows, we validated the theoretical expectation for multi-lane roadways. We found little deviation of the analytical result for multi-lane scenarios, but significant deviations for a single-lane. This can be explained by vehicle platooning. We demonstrate the efficiency of the transverse hopping mechanism for a congestion-warning application in a microscopic traffic simulation scenario. Messages are created on an event-driven basis by equipped vehicles entering and leaving a traffic jam. This application is operative for penetration levels as low as 1%.

Index Terms—Inter-vehicle communication, connectivity, vehicular ad-hoc networks, traffic simulation

I. INTRODUCTION

INTER-VEHICLE communication (IVC) based on wireless communication among vehicles is widely regarded as a promising application for novel transportation services with applications in traffic safety, advanced traveler information and driver assistance systems [1]–[7]. In contrast to conventional communication channels which operate with a centralized broadcasting concept via radio or mobile-phone services, vehicular ad hoc networks (VANETs) provide a *decentralized approach* that does not rely on public infrastructure. Since direct wireless communication has a limited range of, typically, a few hundreds of meters, the effectiveness of potential applications depends crucially on the market penetration. The percentage of equipped vehicles is expected to remain well

below 10% for the next years, so any realistic concept is required to operate at penetration levels of a few percent.

Wireless local area networks based on IEEE 802.11 have shown broadcast ranges between 200 and 500 m in automobile applications [8], [9]. However, VANETs have several characteristics that distinguish them from other ad hoc networks. Among those is the potential change in the node distribution due to the movement of sender/receiver vehicles. For this reason, dedicated protocols and transmission standards are intensively investigated [10]–[13]. Detailed network simulation tools have also been applied to VANET in order to examine the influences of environmental conditions and node mobility on connectivity [14], [15]. For an overview of the technical architectures and protocols, and current research projects and consortia initiatives, we refer to the survey of Hartenstein and Laberteaux [16].

In most automobile applications, it is necessary to carry messages over distances that are significantly longer than the device’s broadcast range meaning that, in general, several equipped vehicles acting as relays are necessary to transport the message to the final destinations. Depending on the application and the specific IVC strategy, a certain minimum percentage of equipped cars is necessary for operation. This characterizes the fundamental problem of a *market penetration threshold*. There are two basic strategies of message propagation:

- (1) A message can be passed backwards to the following equipped vehicle, which then passes it to the next equipped vehicle, and so on, resulting in a node connectivity via multiple hops. We call this *longitudinal hopping*, as the message always propagates parallel to the travel direction of the first sender and the last receiver.
- (2) A sender may transfer a message to a vehicle driving in the opposite direction. This vehicle can store the message and continuously broadcast it for a certain period of time, while physically transporting the message upstream. Although this message is of no use for the relay vehicle, it might eventually be received by an equipped vehicle driving in the original direction by a second transversal hop. We therefore call this strategy “store and forward”, or *transversal hopping*.

The longitudinal hopping mode has the advantage of virtually instantaneous message transmission, so the transmitted information is always up-to-date. However, the longitudinal communication chain typically fails when the broadcast range r is of the same order or smaller than the typical distance

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between equipped vehicles. For realistic broadcast ranges of a few hundreds meters the penetration threshold is about 10% for typical densities on freeways [17]. This restriction can be overcome by the transversal hopping mode, for which the successful transmission is only a matter of time, but the information may be already obsolete when it finally arrives. The direct connectivity is determined mainly by the broadcast range, the traffic density, the market penetration of IVC vehicles and the distribution of equipped vehicles in the traffic stream. The traffic density characterizes the global number of vehicles, but varies strongly in congested traffic conditions. We will therefore focus on the influence of the traffic dynamics on the connectivity of the ad-hoc network of equipped cars. This delay-based connectivity is most useful for traveler traffic information services rather than safety applications.

Previous work on connectivity has been mainly focused on instantaneous connectivity. In the literature, the multiple hopping of messages in longitudinal direction has been studied by several authors [17]–[21]. Due to the inherent complexity of mobile ad hoc communication within the traffic stream, realistic simulations are also essential for validating the analytical models and testing IVC use cases [11], [17], [22]–[24]. Fewer investigations considered the transverse propagation process by means of simulation [25], [26]. To our knowledge, no analytical models have been proposed or investigated for this type of IVC message transport.

In this paper, we will consider the transverse message propagation mode in which messages are transported by vehicles traveling in opposite direction. The system and the relevant transmission processes are described in Sec. II. In Sec. III, we will derive analytical probability distributions for message transmission times and related propagation speeds, assuming a Poissonian distance distribution between equipped vehicles. By means of microscopic traffic simulations combining communication and bi-directional traffic flows, we test the theoretical predictions with numerical results for single and multi-lane roadways in Sec. IV. In the subsequent section, we simulate an incident scenario and consider the propagation of information on shock-fronts and travel times to vehicles in the upstream direction, and discuss future applications in traveler information and “traffic-adaptive” cruise control systems. In Sec. VI, we conclude with a discussion of our findings.

II. MESSAGE TRANSPORT VIA THE OPPOSITE DRIVING DIRECTION

In this section, we consider a concept of inter-vehicle communication, using equipped vehicles in the opposite driving direction as relays. These vehicles transport the message to a destination region where it is delivered back to vehicles in the original driving directions. Thus, the wireless communication part of the mechanism considered consists of exactly two “transverse message hops” across driving directions.

In the following, a message is considered to be useful if it reaches an equipped vehicle driving in the same direction as the vehicle creating the initial message before the receiver reaches the position of the original message. For example, if the initial message contains information about a locally

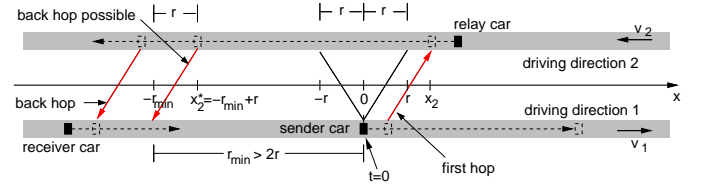


Fig. 1. Illustration of message transport via the opposite driving direction. At time $t = 0$, a car in direction 1 generates a message and starts broadcasting it. The message is transported forward at speed v_1 until it will be received by an equipped car moving at speed v_2 in direction 2. The relay car transports the message at least up to the position $-r_{\min} + r$ (the destination region) before broadcasting it again. Finally, the message is received by an equipped car in direction 1. Transversal distances are neglected when determining the communication range r .

detected traffic jam, this information is useful only for vehicles sufficiently upstream of this location and driving in the same direction. Specifically, we define as *destination region* all locations in this direction that are upstream of the original source by at least the distance r_{\min} (typical values for r_{\min} are of the order of 1 km). Message transmission by the transverse hopping strategy can be considered as a three-stage process:

- (1) After a message is generated by an equipped vehicle, the message is continually broadcasted by this vehicle for a certain time. It is received by a relay vehicle in the opposite driving direction after a time interval τ_1 .
- (2) The message is then transported by the relay vehicle. After a time interval τ_2 (with respect to the initial generation of the message) this vehicle's broadcast range starts intersecting the destination region and the vehicle starts broadcasting the stored message.
- (3) At time τ_3 (with respect to the initial generation time) the message is received by an equipped vehicle in the destination region. The relay vehicle continues broadcasting until the message is considered to be obsolete (the criteria for this condition will not be considered here).

Because of the complex traffic dynamics, the time intervals τ_i (with $i = 1, 2, 3$) can be considered as *stochastic variables*.

III. ANALYTICAL MODEL

For the description of the information propagation via the opposite driving direction, we consider a bi-directional free-way or arterial road (cf. Fig. 1). In both directions, we assume essentially homogeneous traffic flows that are characterized by the lane-averaged velocities v_1 and v_2 , and the total densities ρ_1 and ρ_2 , respectively. Furthermore, we assume a global market penetration α such that the relevant density variables are the *partial densities of equipped vehicles*, $\lambda_1 = \alpha\rho_1$ and $\lambda_2 = \alpha\rho_2$, respectively. Communication between equipped vehicles is possible within a limited broadcast range r . Within this range, the communication is assumed to be error free and instantaneous. In Sec. III-D, we will relax this assumption and derive an analytical model for the more realistic case of distributed broadcast ranges [15], [16]. By virtue of the assumed constant velocities, the first relay vehicle having received the message will also be the first vehicle reaching

the destination region, so we will ignore message reception and transport by subsequent relay vehicles.

Let us consider a message that is generated by an equipped vehicle driving in direction 1 at position $x = 0$ and time $t = 0$. As illustrated in Fig. 1, we use the same coordinate system for both directions such that driving direction 1 is parallel and direction 2 anti-parallel to the x -axis. We start with the spatial distribution of vehicles in the opposite driving direction. We assume that the distances between *equipped* vehicles are i.i.d. exponentially distributed stochastic variables which is a good approximation as long as the partial densities λ_1 and λ_2 are small [17]. Because of the implied Poisson process, the same distribution applies to the distance between *any* given location x and the next equipped vehicle.

A. First Transversal Hop

First, we investigate the statistical properties of the time τ_1 of the first transversal hop to a relay vehicle in the opposite direction 2. Since only the *first* transmission of a message is relevant, the reference point for finding such a vehicle is given by the “best case”. In this case, the initial position of the relay is at $x_2 = -r$, i.e., at the limit of the transmission range in the desired direction. The actually used transmitter vehicle is the equipped vehicle located next to this point when going in the direction of positive x . According to the above assumptions, its initial position X_2 is a stochastic variable that is defined by the probability density

$$f(x) = \lambda_2 e^{-\lambda_2(x+r)} \Theta(x+r). \quad (1)$$

Here, the Heaviside function $\Theta(x)$ (which is equal to 0 for $x < 0$ and equal to 1 for $x \geq 0$) serves as a cut-off for receiver positions at $x < -r$, which are too remote. Notice that a “conventional” exponential distribution would result when starting the search for a relay vehicle at $x = 0$, i.e., setting $r = 0$.

An instantaneous transversal hop of the generated message to the relay vehicle on roadway 2 is possible if it is within the broadcast range, $-r \leq X_2 \leq r$. This happens with probability

$$P_1(0) = \int_{-r}^r f(x) dx = 1 - e^{-2\lambda_2 r}. \quad (2)$$

Otherwise, a finite time interval τ_1 is needed before the relay vehicle comes within communication range (cf. Fig. 1). Since the relative velocity between vehicles in opposite driving directions is $v_1 + v_2$, this time depends on the initial position X_2 according to $\tau_1 = (X_2 - r)/(v_1 + v_2)$. Therefore, a successful first hop is possible at time τ or before, if the initial position of the receiving vehicle is in the interval $X_2 \in [-r, x_2(\tau)]$ where $x_2(\tau) = (v_1 + v_2)\tau + r$. Consequently, the distribution function for τ_1 (the probability for a successful first hop at time τ_1 or earlier) is given by

$$P_1(\tau) = \int_{-r}^{x_2(\tau)} f(x) dx = 1 - e^{-\lambda_2[2r + (v_1 + v_2)\tau]}. \quad (3)$$

This probability is shown in Fig. 2(a) for different market penetrations α influencing the partial density of equipped cars, $\lambda_2 = \alpha\rho_2$.

B. Possibility of Hopping to Original Driving Direction

Now we investigate the distribution of the time interval τ_2 after which the message is available for retransmission to the original driving direction 1, i.e., the relay vehicle starts broadcasting the message. It should occur at a location x_3 that, for this direction, is sufficiently upstream of the original message source, i.e., $x_3 \leq -r_{\min}$. The transmission becomes possible when the relay vehicle passes the position $x_2^* = -r_{\min} + r$. Because of the assumed constant velocities v_2 in negative x -direction, the position of the relay vehicle depends on time according to $X_2(\tau) = X_2 - v_2\tau$, where the initial position X_2 is distributed according to the density (1). Notice that, by virtue of the constant velocity v_2 , the distribution function of the position of the relay vehicle is shifted uniformly in time. Starting directly from its definition, the distribution function for τ_2 is given by $P_2(\tau) = P(X_2(\tau) < x_2^*) = P(X_2 < v_2\tau + x_2^*) = F(v_2\tau + x_2^*)$, where $F(x)$ is the cumulative distribution function of the probability density (1). Inserting this equation results in

$$P_2(\tau) = \int_{-r}^{v_2\tau + x_2^*} f(x) dx = \Theta\left(\tau - \frac{r_{\min} - 2r}{v_2}\right) \left(1 - e^{-\lambda_2(2r + v_2\tau - r_{\min})}\right). \quad (4)$$

Since the message dissemination depends on the relay car in direction 2 only, the probability (4) is independent of the average vehicle speed v_1 in direction 1 and the time τ_1 of the first hop. Figure 2(b) shows the cumulative distribution $P_2(\tau)$ for different market penetrations α .

C. Successful Message Transmission to Original Driving Direction

Finally, we calculate the distribution of the time τ_3 for the first successful message receipt by a vehicle traveling in the destination region, i.e., in direction 1 at a position $x \leq -r_{\min}$. In general, this time differs from τ_2 since, at this time, the message can be received only by a vehicle located exactly at $x = -r_{\min}$. For $t > \tau_2$, the average arrival rate of equipped vehicles in the destination region, reaching the range of the available message, is given by the relative flow $\lambda_1(v_1 + v_2)$. For the assumed Poissonian process, the time interval $T = \tau_3 - \tau_2$ for the arrival of the first receiver vehicle is exponentially distributed with the probability density

$$f_T(\tau) = \Theta(\tau) \lambda_1(v_1 + v_2) e^{-\lambda_1(v_1 + v_2)\tau}. \quad (5)$$

Furthermore, by means of the same Poissonian assumption, the stochastic variables T and τ_2 are independent, and the distribution function $P_3(\tau) = P(\tau_3 < \tau)$ for the sum $\tau_3 = \tau_2 + T$ is, therefore, given by the convolution

$$P_3(\tau) = \int_0^\infty f_T(t) P_2(\tau - t) dt. \quad (6)$$

Here, the Heaviside function of Eq. (5) has been used to limit the lower boundary of the integral. Inserting the density

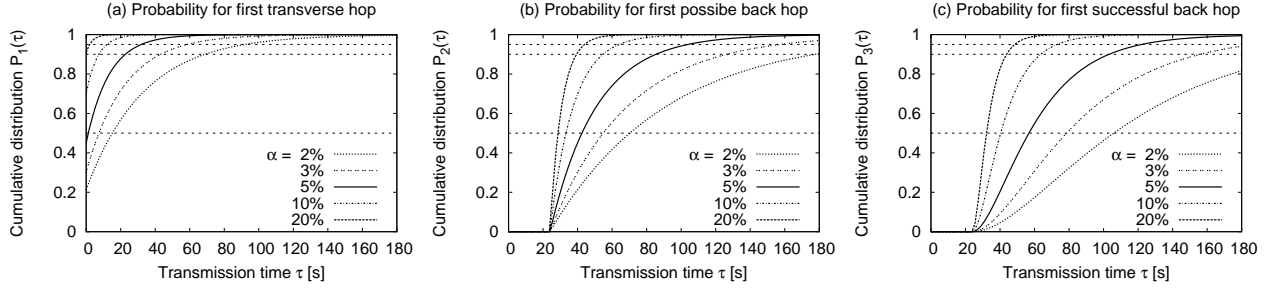


Fig. 2. Statistical properties of the characteristic times for the transverse hopping mechanism. The plots show (a) the distribution functions for the time interval τ_1 between message creation and the first transverse hop, (b) the time interval τ_2 after which the message becomes available in the destination region, and (c) the time τ_3 of the first reception by a car in the destination region. The market penetration level α is varied, while the other parameters are kept constant (broadcast range $r = 200$ m, traffic densities $\rho_1 = \rho_2 = 30/\text{km}$, average speeds $v_1 = v_2 = 90$ km/h).

function f_T and expression (4) results in

$$P_3(\tau) = \Theta(\tau - \tau_{\min}) \lambda_1 (v_1 + v_2) \int_0^{\tau - \tau_{\min}} e^{-\lambda_1 (v_1 + v_2) t} \left(1 - e^{-\lambda_2 v_2 (\tau - \tau_{\min} - t)}\right) dt \quad (7)$$

where the minimum transport time for a successful complete transmission is given by $\tau_{\min} = (r_{\min} - 2r)/v_2$. Notice that the function $\Theta(\tau - \tau_{\min})$ ensures that the upper integral limit is larger than the lower one. Finally, basic integration and introduction of the abbreviation $\tilde{\lambda}_1 = \lambda_1 \frac{v_1 + v_2}{v_2}$ yield the solution

$$P_3(\tau) = \Theta(\tau - \tau_{\min}) \left(1 - \frac{\tilde{\lambda}_1}{\tilde{\lambda}_1 - \lambda_2} e^{-\lambda_2 x_e(\tau)} + \frac{\lambda_2}{\tilde{\lambda}_1 - \lambda_2} e^{-\tilde{\lambda}_1 x_e(\tau)}\right), \quad (8)$$

where $x_e(\tau) = v_2(\tau - \tau_{\min}) = v_2\tau - r_{\min} + 2r$ denotes the part of the destination region that intersects (or has been intersected by) the range of the relay vehicle for the “best case”. In case of identical traffic conditions in both driving directions (i.e., $v_1 = v_2 = v$ and $\lambda_1 = \lambda_2 = \lambda$), we have $\tilde{\lambda}_1 = 2\lambda$, resulting in the more intuitive expression

$$P_3(\tau) = \Theta\left(\tau - \frac{r_{\min} - 2r}{v}\right) \left[1 - e^{-\lambda(2r + v\tau - r_{\min})}\right]^2 \quad (9)$$

which is shown in Fig. 2(c). Note that the quadratic term in Eq. (9) reflects the fact that two encounters of equipped vehicles are needed for propagating a message by means of transverse message hopping.

For an evaluation and discussion of the obtained results, we consider characteristic quantities of the obtained probability distributions which, for simplicity, are given for the symmetric case reflected by Eq. (9). A suitable measure for assessing the performance of the proposed communication scheme are the quantiles $\tau_3^{(q)}$, indicating the total communication time that is only exceeded by the fraction $1 - q$ of all message transports (cf. the horizontal lines in Fig. 2). The defining condition $P_3(\tau^{(q)}) = q$ leads to

$$\tau_3^{(q)} = \frac{r_{\min} - 2r}{v} + \frac{\ln(1 - \sqrt{q})}{\lambda v}. \quad (10)$$

Table I lists the values for the median ($q = 0.5$), the 90% and the 95% quantiles, respectively, using the parameters considered in Fig. 2. For example, even for a penetration level as low as 2%, half of all message transports have been completed (which is defined by the fact that a car in the destination region at least 1 km upstream of the information source has received the message) in 106 s or less. Moreover, 90% of the propagated messages have been completed in 222 s or less and only for 5% of all messages the communication time exceeds about 5 min, after which one could consider the information as obsolete.

Furthermore, the Table I lists the expected averages, the analytical values of which are given for the symmetric case by

$$\langle \tau_2 \rangle = \frac{r_{\min} - 2r}{v} + \frac{1}{\lambda v}, \quad \langle \tau_3 \rangle = \frac{r_{\min} - 2r}{v} + \frac{3}{2\lambda v}. \quad (11)$$

The latter quantity leads us to the definition of the average speed of information dissemination:

$$v_p = \frac{r_{\min}}{\langle \tau_3 \rangle}. \quad (12)$$

We will discuss the relevance of this quantity for the spread of information in traffic flows in Sec. V below.

D. Distributed communication ranges

In the previous section, we have derived the probability $P_3(\tau)$ for a successful communication after the time interval τ assuming a fixed communication range with 100% connectivity for distances less than r , and no communication for larger distances. In order to assess the errors made by this somewhat unrealistic assumption, we will now derive the probability $P_3(\tau)$ for distributed direct communication ranges given by the density function $g(r)$. Assuming a perfect correlation of the ranges for the first and the second hop (which arguably is the “worst case” in terms of deviations to the simpler model (8)), the probability P_3^{dist} for distributed communication ranges is the weighted average of the probability $P_3(\tau|r)$ for a given direct communication range r weighted with the probability density:

$$P_3^{\text{dist}}(\tau) = \int_0^\infty dr g(r) P_3(\tau|r). \quad (13)$$

This expression can be analytically integrated, e.g., for uniformly, Gaussian, or exponentially distributed ranges. For the exponential distribution, $g(r) = \lambda_r e^{-\lambda_r r}$ for $r \geq 0$, we obtain

$$P_3^{\text{dist}}(\tau) = e^{-\lambda_r r_0} \left[1 - \frac{\tilde{C}_1 \lambda_r}{\lambda_r + 2\lambda_2} e^{-2\lambda_2 r_0} + \frac{\tilde{C}_2 \lambda_r}{\lambda_r + 2\lambda_1} e^{-2\lambda_1 r_0} \right] \quad (14)$$

where

$$\tilde{C}_1 = \frac{\tilde{\lambda}_1}{\tilde{\lambda}_1 - \lambda_2} e^{\lambda_2(r_{\min} - v_2 \tau)}, \quad (15)$$

$$\tilde{C}_2 = \frac{\lambda_2}{\tilde{\lambda}_1 - \lambda_2} e^{\tilde{\lambda}_1(r_{\min} - v_2 \tau)}, \quad (16)$$

$$r_0 = \max \left[0, \frac{1}{2}(r_{\min} - v_2 \tau) \right]. \quad (17)$$

E. Discussion of Model Assumptions

Let us finally discuss the implications of the three main model assumptions: (i) exponential headway distributions, (ii) homogeneous traffic flow, and (iii) fixed vs. distributed communication ranges. When relaxing the assumption of an exponential distribution (which is not valid for high traffic densities and, simultaneously, high market penetration levels), the lines of the model's derivation remains valid as long as the gaps between vehicles remain uncorrelated. However, it is to be expected that the resulting integrals are more cumbersome to solve or cannot be analytically solved at all. Nevertheless, we will show by means of traffic simulations in Sec. IV that the results are remarkably robust with respect to violations of the Poisson assumption.

The assumption of homogeneous traffic flow implies a more serious restriction as, after all, traveler information about traffic congestion is considered to be a primary application of IVC. Here, traffic instabilities will clearly lead to nonhomogeneous traffic flows. This point can be clarified by looking at the derivation. According to the convolution formula (6), the main result (8) depends on the probability distribution $P_2(\tau)$ for the time interval of the first availability of a message, and the probability density f_T for the arrival time of the first destination vehicle. According to Eq. (4), $P_2(\tau)$ depends on the traffic situation in the *opposite* direction only, while f_T as given by Eq. (5) depends on the traffic flow in both directions, but only in the *target* region. Consequently, the analytical result remains valid even in case of traffic congestion in the source region (or if the message has been created and broadcasted by a standing vehicle), as long as there are no traffic jams in the opposite direction *and* in the destination region. If traffic is congested in the destination region, the system generally performs better than predicted. Only if there is congested traffic in the *opposite* direction, the message propagation is significantly disturbed and the analytical results are no longer applicable. However, this is not a serious scenario as traffic information is important for the congested lane rather than for the lane with freely flowing traffic.

In order to determine if the assumption of stochastic communication ranges significantly changes the results compared to a fixed communication range, we plot the corresponding

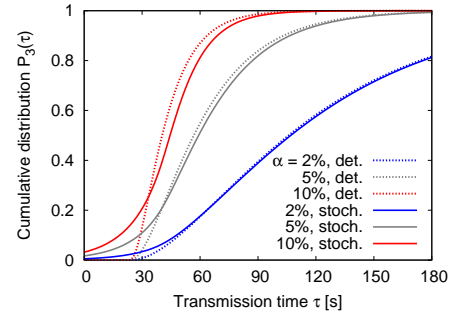


Fig. 3. Transmission probability $P_3(\tau)$ as a function of the transmission time τ for a fixed direct communication range $r = 200$ m (thin lines), and for exponentially distributed ranges with the same expectation value $E(r) = 200$ m, and 100% correlation between the two hops. The values for ρ_1 , ρ_2 , v_1 , v_2 , and r_{\min} are the same as in Fig. 2.

TABLE I
CHARACTERISTIC QUANTITIES FOR THE EFFICIENCY OF MESSAGE DISSEMINATION VIA THE OPPOSITE DRIVING DIRECTION. THE OTHER PARAMETERS ARE IDENTICAL TO THE SCENARIO ILLUSTRATED IN FIG. 2.

α	$\langle \tau_2 \rangle$ (s)	$\langle \tau_3 \rangle$ (s)	$\tau_3^{(0.5)}$ (s)	$\tau_3^{(0.9)}$ (s)	$\tau_3^{(0.95)}$ (s)	v_p (km/h)
1%	157	224	188	420	514	16.1
2%	90.7	124	106	222	269	29.0
3%	68.4	90.7	78.6	156	187	39.7
5%	50.7	64.0	56.7	103	122	56.3
10%	37.3	44.0	40.4	63.6	73.0	81.8
20%	30.7	34.0	32.2	43.8	48.5	105
50%	26.7	28.0	27.3	31.9	33.8	128

expressions (8) and (14) for $\lambda_r = 1/r$, i.e., assuming the same average communication range in both models. Figure 3 shows that for the practically relevant regimes of low percentages α and high values of P_3 , the differences (of the order of one percent) are negligible. We will therefore refer to the simpler fixed-range model (8) in the rest of this paper. Note also that the stochastic model has a higher connectivity for small values of τ but lower connectivities later on with a crossover at about $P_3 = 0.25$. This is to be expected since additional stochasticities typically smear out distributions.

IV. MICROSCOPIC SIMULATION RESULTS

In this section, we compare the predictions of the analytic model of Sec. III to simulation results obtained by a microscopic traffic simulator that has been extended to support inter-vehicle communication. Our principal interest is to check for the reliability and robustness of our theoretical results based on the assumption of exponentially distributed vehicles. How much error is introduced by neglecting minimum safe distances between vehicles and/or inhomogeneities of traffic flow, which clearly exist in real traffic situations?

We have carried out a multi-lane traffic simulation of a 20 km freeway stretch with two independent driving directions and one to four lanes per direction. The simulator uses the *Intelligent Driver Model* [27] as a simple, yet realistic, car-following model, and the general-purpose lane-changing algorithm MOBIL [28]. A simplified demo version of this simulator can be run interactively on the web [29], [30]. In order to introduce a minimal degree of driver heterogeneity, the

desired velocities of the driver-vehicle units have been chosen Gaussian distributed with a standard deviation of 18 km/h around a mean speed of $v_0 = 120$ km/h which is typical for a German freeway with speed limits. The other model parameters turned out not to be relevant for the resulting statistics. Remarkably, this was even true for the time headway parameter of the car-following model, although this parameter directly influences the gaps between vehicles. Furthermore, we used open boundary conditions with a constant inflow $Q_{in} = 1200$ /h/lane at the upstream boundary. Note that the traffic density is a result of the traffic dynamics. To compare the simulation results with the analytical results, we determined the traffic densities (ρ_1 and ρ_2) and average speeds (v_1 , v_2) for both driving directions by measurement via a simulated loop detector.

For the purpose of this simulation study, we extended the traffic simulator by a communication module. After selecting the IVC vehicles among all vehicles randomly according to the penetration level α , each IVC vehicle was equipped with a “communication device” which allows for generating, sending, receiving, and storing messages. Messages were generated on an event-driven basis that depended on the considered IVC application (cf. Sec. V below). Here, test messages were generated whenever an IVC vehicle passed a “reference landmark” at $x = 10$ km. Furthermore, we assumed a perfect communication channel: Messages were broadcasted continually and vehicles in the opposite driving direction received the messages instantaneously and without any transmission errors as soon as they were within the communication range r . Transversal distances were neglected when determining whether a vehicle was within the broadcast range. The message transport was simulated exactly as described at the beginning of Sec. II.

Each simulation typically simulated several hours of real time and generated about 100 000 message transmissions, serving as a solid basis for the statistical analysis. Figure 4 shows the simulated and the analytical distributions for the transmission times τ_1 , τ_2 and τ_3 for one lane per direction and four different penetration levels α . For low penetration levels, we have observed a good agreement between the simulated and analytical results while, for higher penetration levels of $\alpha = 6\%$ and 10% , the median transmission times in the simulations were up to 10% higher than analytically predicted.

As a quantitative measure for the overall discrepancy, we introduce the uncertainty measure

$$U = \frac{\sum_i (x_i - \hat{x}_i)^2}{\sum_i (x_i - \bar{x})^2}. \quad (18)$$

Here, the sum runs over all message transmission times i up to the analytical 95% quantile, x_i and \hat{x}_i denote the observed and analytical values for the cumulative distribution of the relevant time, respectively, and \bar{x} is the arithmetic mean of the x_i values. For the simulations shown in Fig. 4, we have obtained the values $U = 0.006, 0.013, 0.030$ and 0.084 for $\alpha = 2\%, 4\%, 6\%$, and 10% , respectively. This finding confirms that the agreement is best for low penetration levels.

When looking at the actual vehicle positions observed in the simulations, the reason for the deviations at higher penetration levels becomes obvious. Figure 5 shows snapshots



Fig. 5. Screenshots of one driving direction in a single-lane traffic simulation at three locations showing the clustering of vehicles in platoons due to distributed desired velocities in combination with the lack of overtaking possibilities. For the purpose of illustration, the market penetration of equipped (labeled as yellow) vehicles was set to 20%.

of the simulator for one driving direction with a single lane. Due to the distributed desired velocities and the lack of overtaking possibilities, vehicles that initially enter the road with identical headways tend to cluster behind slower vehicles. At $x = 10$ km, i.e., at the point of message generation, this results in significant vehicle platoons. Both the cluster-forming process and the minimum safety gap between the vehicles make the arrival process significantly *non-Poissonian*. In particular, the large gaps between the clusters lead to larger transmission times in the simulations as compared to our previous analytical results. Nevertheless, for sufficiently low penetration levels, the average distance between equipped vehicles becomes comparable to the typical cluster size. As a consequence, the Poissonian assumption can be satisfied for the *IVC equipped vehicles*, even if this is not the case for *all* vehicles. This explains the good agreement of the theoretical prediction for *low* penetration levels and shows that the analytical model of Sec. III is quite robust with respect to violations of its assumptions.

Finally, Figure 6 shows a comparison between the analytical model and simulated message propagation for different numbers of lanes per direction, while keeping the penetration level at a constant value $\alpha = 5\%$. It turns out that the discrepancies vanish almost completely, when there is more than one lane. While the uncertainty measure was $U = 0.024$ for one lane, we observed $U < 0.0004$ for two to four lanes. In terms of cumulative distribution functions, the error is less than $\sqrt{U} = 2\%$. The same applies to the relative error of the median (or other quantiles) of the message transport times. The primary reasons for this good agreement are the overtaking opportunities preventing the build-up of large vehicle platoons, at least, if no traffic breakdowns occur. Generally, we found that the error decreases with an increasing number of lanes and with a decreasing penetration level.

V. IVC FOR JAM FRONT DETECTION AND TRAVELER INFORMATION

In this section we demonstrate the efficiency of the transverse hopping mechanism by simulating the propagation of messages regarding traffic conditions that are created by vehicles entering and/or leaving a traffic jam. The fronts of the jam can be autonomously detected by individual vehicles based on the decrease or increase in the average driving speed [5], [31]. We have simulated a bi-directional roadway with two lanes in

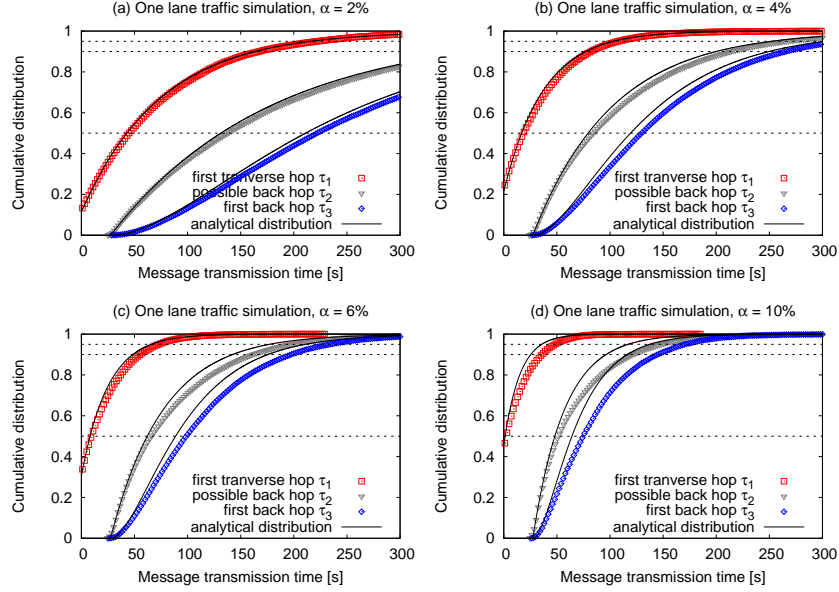


Fig. 4. Simulated vs. analytical results for the statistical properties of the characteristic communication times for four market penetrations on a road with one lane per direction (cf. Fig. 5). The broadcast range r is 200 m. The average speed v (density ρ) of both driving directions was determined from the simulations approximately to 78.8 km/h (15.2/km).

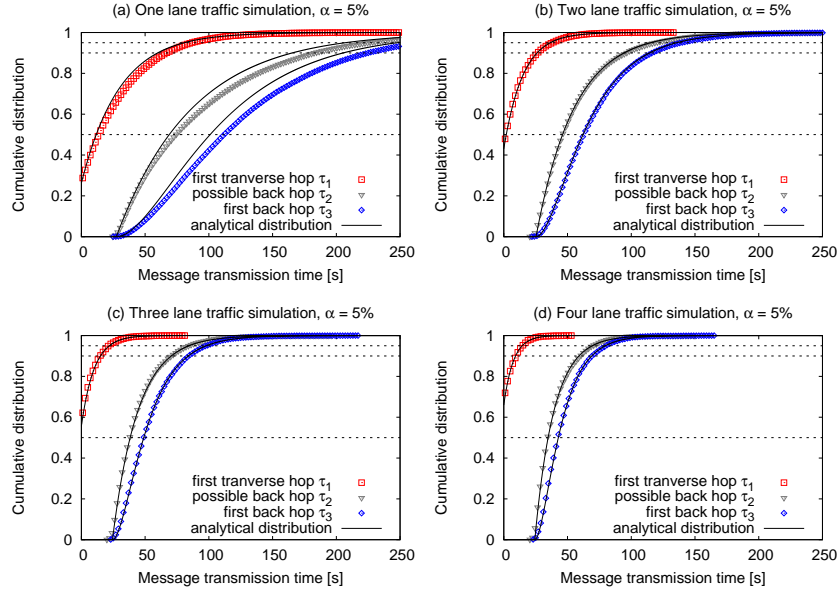


Fig. 6. Simulated and analytical results for the statistical properties of the characteristic communication times, studying roads with one to four lanes per direction. The market penetration $\alpha = 5\%$ and the broadcast range $r = 200$ m are kept constant while the average speed and density of both driving directions were determined from the simulations ($v = 78.8$ km/h, $\rho = 15.2$ /km for 1 lane; $v = 85.4$ km/h, $\rho = 28.1$ /km for 2 lanes; $v = 88.7$ km/h, $\rho = 40.6$ /km for 3 lanes; $v = 88.8$ km/h, $\rho = 53.6$ /km for 4 lanes).

each driving direction and with a bottleneck at $x = 10$ km in one driving direction. While traffic is free in one direction, traffic becomes congested in the other due to an increasing demand that exceeds the capacity at the bottleneck (peak-hour scenario with time-varying boundary conditions). The resulting spatiotemporal contour plot of the average speed is shown in Fig. 7. Furthermore, the trajectories of the IVC-equipped vehicles are displayed by solid and dotted lines depending on the driving direction. Notice that the market

penetration is only 1%.

In our simulation, the breakdown of traffic flow occurs approximately after 14 min at the bottleneck location. For the purpose of illustration, we discuss the involved processes by means of two equipped vehicles which are marked in Fig. 7 by thicker lines labeled as “1” and “2”. First, the upstream end of the jam is detected by the decrease in speed (corresponding to the change in the slope of the trajectory) which is labeled as “1a” in the space-time diagram. Second, the same

vehicle records the downstream end of the traffic jam while accelerating to its desired speed, thereby creating the message “1b”. The same processes apply to car 2 labeled as “2a” and “2c”. These messages are received by equipped vehicles in the opposite driving direction within the communication range, denoted as events “1c”, “2b” and “2d”, respectively. Finally, the messages are transported in upstream direction and broadcasted, until they are received by vehicles in lane 1 at later times. In Fig. 7, this corresponds to the intersection of trajectories of relay vehicles (thicker points) with a solid-line trajectory.

In order to be useful for traveler information services, the messages have to propagate faster than the fronts of the traffic jam. It is known from empirical observations that the head of a traffic jam (the “downstream jam front”) is either fixed at the bottleneck or moves upstream with a characteristic speed of 15 ± 5 km/h. Moreover, possible propagation velocities for the jam front moving in upstream direction range from 0 to about 18 km/h, depending on the current inflow [32]. Note that in case of dissolving congestion, the upstream jam front can also propagate in *downstream* direction. In the simulation example shown in Fig. 7, the upstream front of the traffic jam moves in upstream direction with a speed of about 10 km/h, while the downstream front is fixed at the (stationary) bottleneck.

In Eq. (11) of Sec. III, we have defined the average speed of information dissemination v_p with respect to the first vehicle that received this message in the target region located upstream of the source by at least a distance r_{\min} . Nevertheless, the same message is also useful for later vehicles in the destination region, at least for some time. Since, during this time, the message is transported by the relay vehicle with a speed larger than v_p , Eq. (11) denotes the *worst case*. For example, the messages “1a” and “1b” referring to the situation at $t = 15$ min are received by several vehicles within the following 2 to 3 minutes. From the end user’s point of view, a “usability measure” could consider the up-to-dateness of the information about the upstream and downstream jam fronts at a given distance from the jam, e.g., 1 km. At this distance, the messages 1a and 1b are the most recent ones for three vehicles (entering the section shown in Fig. 7 at times between 15 min and 16.5 min). For these vehicles, the “age” of the upstream information 1a lies between 2.3 min and 3.1 min. The downstream information 1b is even more recent. For the set of 9 vehicles located between message 1a and 2a, the average message age is 2.5 min.

Note that one can also take into account that a traffic management center or the police may send out “public” messages about the current traffic state, roadwork or incident conditions, complementing messages created by autonomous vehicles. Representing this by a standing vehicle in the shoulder lane for the initial message broadcast, this will not change the further transmission times. A standing vehicle would correspond to a horizontal trajectory in Fig. 7.

Finally, we point out that messages regarding jam front positions are up-to-date as long as the *collective* traffic dynamics does not change significantly. While the downstream jam front is fixed at the bottleneck (and therefore easy to predict

at later times), the moving upstream front can be estimated with an accuracy of several 100 m. However, data-fusion of *several* messages and model-based prediction can be used to reduce these errors dramatically [33]. For a quantitative analysis of such a vehicle-based jam-front detection, we refer to Refs. [26], [33].

VI. DISCUSSION AND CONCLUSIONS

Inter-vehicle communication (IVC) based on vehicular ad hoc networks (VANETs) is a promising and scalable concept for exchanging traffic-related information among vehicles over relatively short distances. Advanced traveler information systems are recognized as an important application of this decentralized approach in the first deployment phase, as IVC will provide local and up-to-date traffic information. Apart from the drivers appreciating such reliable and up-to-date traffic information, future driver assistance and safety systems may benefit from IVC as well, for example, by issuing warnings when a traffic jam or an accident is several hundred meters ahead.

Adaptive cruise control can automate the braking and acceleration of a car. Processing of non-local information received via IVC could help future “traffic-adaptive” cruise control systems to anticipate the traffic situation and therefore to automatically adapt the driving style to it, increasing driving comfort, safety and traffic performance [5], [7]. Notice that the sensor technology needed for driver assistance systems can be used to automatically produce messages on an event-oriented basis.

However, like all technologies relying on local communication, IVC faces the “penetration threshold problem”. Thus, the system is effective only if there is a sufficient number of communication partners to propagate the message between equipped cars. Therefore, it is crucial to assess the feasibility of different communication variants in terms of the necessary critical market penetration.

In this paper, we have investigated this aspect both, analytically and by simulation for a basic strategy of message dissemination by “transverse hopping”. In this mode, equipped vehicles in the opposite driving direction are used to transport the messages serving as relays. Our results indicate that the transverse hopping mechanism is favorable in the first stages of the deployment of an cooperative IVC system, since it is already effective for market penetration levels as low as 1-2%.

In order to gain more insights into the factors influencing the reliability and effectiveness of the “store-and-forward” message propagation in IVC systems, we derived analytical expressions for communication delay times. An important result of this paper is that the analytically derived statistics have been confirmed by extensive traffic simulations. Although the main assumptions made in deriving these models – homogeneous traffic and exponentially distributed inter-vehicle distances – are normally not perfectly met in real traffic flow, it turns out that the theoretical results are remarkably robust with respect to violations of the models’ assumptions. In particular, for multi-lane traffic and penetration levels below 5%, the errors are typically a few percent only. Notice that, in turn,

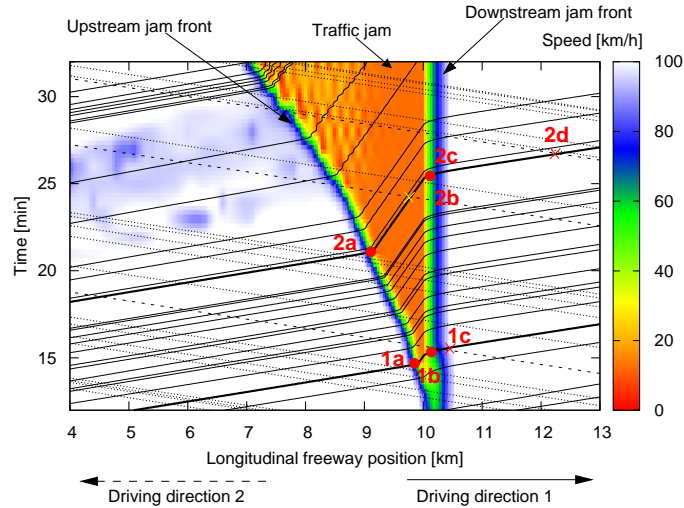


Fig. 7. Space-time diagram of a scenario with traffic congestion. The contour plot shows the average speed of all vehicles in driving direction 1. Trajectories of equipped vehicles (only 1% of all vehicles) are displayed by solid and dashed lines, depending on the driving direction. Points indicate the generation of messages when detecting the shock fronts of the jam and crosses indicate message transmissions.

the analytical expressions may also serve to test or validate new implementations of communication modules in traffic simulation software.

One may wonder why the idealized assumptions work particularly well in multi-lane traffic which clearly is more complex than single-lane traffic. This is mainly caused by the possibility to change lanes and overtake thereby dissolving platoons behind slow vehicles. The positional correlations implied by platoons, in turn, are the main cause why the assumption of the Poissonian positional distribution is not satisfied. Other kinds of complexity, however, lower the reliability of this assumption. For example, in stop-and-go traffic or at intersections the overall traffic flow varies strongly (typically on scales of 1 min per 1 km). Such non-stationary traffic conditions can only be approximately treated as a superposition of the prevailing traffic densities.

Since our focus was on traffic dynamics rather than on the details of the communication protocols, we generally assumed idealized communication conditions, i.e., instantaneous and error-free transmission below a certain direct communication range and a failure rate of 100% above. However, we have shown analytically that the transmission properties change in a predictable way when assuming more realistic communication conditions. For example, relaxing the assumption of a fixed communication range by treating the communication range as a random variable resulted in little changes (Fig. 3). Furthermore, assuming a complete failure of the communication chain with a probability f simply will reduce the market penetration parameter α by a factor $1 - f$, at least, as long as the failures can be considered to be uncorrelated.

The assumption that a vehicle continuously broadcast messages could be relaxed by considering a periodic broadcast of messages in time intervals τ_t . This would result in two additional convolutions of the relevant transmission time τ_3 with the uniform distribution in the interval $[0, \tau_t]$. This essentially increases the median of the communication time

by $2\tau_t/2 = \tau_t$.

It remains to be shown to which extent other imperfections of real communications (such as delay times for establishing a direct communication, failures due to high relative velocities, or channel conflicts and contention for an increasing penetration level) will influence the results. In any case, the microscopic details of DSRC communication require network simulation software such as ns2.

For demonstration purposes, we considered a congestion-warning application in a complex traffic simulation which is operative for penetration rate as low as 1%. However, IVC is only one building block of a future integrated traffic communication system. As a straightforward next step, including police cars and emergency vehicles into the IVC fleet will lead to a timely production of event-related messages. Furthermore, adding infrastructure-vehicle communication to the system may help to overcome the penetration barrier [34]. This will be particularly economic and efficient when placing the infrastructural communication units near to known bottlenecks, where the necessary sensors for producing event-based traffic messages are already in place.

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REFERENCES

- [1] S. Shladover, G. Polatkan, R. Sengupta, J. VanderWerf, M. Ergen, and B. Bouglér, “Dependence of Cooperative Vehicle System Performance on Market Penetration,” *Transportation Research Record*, vol. 2000, pp. 121–127, 2007.

- [2] L. Wischhof, A. Ebner, and H. Rohling, "Information dissemination in self-organizing intervehicle networks," *IEEE Transactions on Intelligent Transportation Systems*, vol. 6, pp. 90–101, 2005.
- [3] J. Zhang, A. Ziliaskopoulos, N. Wen, and R. Berry, "Design and implementation of a vehicle-to-vehicle based traffic information system," *IEEE Proceedings of Intelligent Transportation Systems*, pp. 473–477, 2005.
- [4] H. Mahmassani, "Dynamic Network Traffic Assignment and Simulation Methodology for Advanced System Management Applications," *Networks and Spatial Economics*, vol. 1, no. 3, pp. 267–292, 2001.
- [5] A. Kesting, M. Treiber, M. Schönhof, and D. Helbing, "Adaptive cruise control design for active congestion avoidance," *Transportation Research Part C: Emerging Technologies*, vol. 16, no. 6, pp. 668–683, 2008.
- [6] S. Kato, S. Tsugawa, K. Tokuda, T. Matsui, and H. Fujii, "Vehicle control algorithms for cooperative driving with automated vehicles and intervehicle communications," *IEEE Transactions on Intelligent Transportation Systems*, vol. 3, no. 3, pp. 155–161, 2002.
- [7] B. van Arem, C. van Driel, and R. Visser, "The impact of cooperative adaptive cruise control on traffic-flow characteristics," *IEEE Transactions on Intelligent Transportation Systems*, vol. 7, no. 4, pp. 429–436, 2006.
- [8] J. P. Singh, N. Bambos, B. Srinivasan, and D. Clawin, "Wireless LAN performance under varied stress conditions in vehicular traffic scenarios," in *IEEE Vehicular Technology Conference*, 2002, pp. 743–747.
- [9] J. Ott and D. Kutscher, "Drive-thru internet: IEEE 802.11b for "automobile" users," in *Proceedings of Infocom 2004, Twenty-third Annual Joint Conference of the IEEE Computer and Communications Societies*, 2004, pp. 362–373.
- [10] G. Korkmaz, E. Ekici, F. Özgüner, and Ü. Özgüner, "Urban multi-hop broadcast protocol for inter-vehicle communication systems," *Proceedings of the First ACM Workshop on Vehicular ad hoc Networks*, pp. 76–85, 2004.
- [11] M. Artimy, "Local Density Estimation and Dynamic Transmission-Range Assignment in Vehicular Ad Hoc Networks," *IEEE Transactions on Intelligent Transportation Systems*, vol. 8, no. 3, pp. 400–412, 2007.
- [12] M. Saito, J. Tsukamoto, T. Umedu, and T. Higashino, "Design and Evaluation of Intervehicle Dissemination Protocol for Propagation of Preceding Traffic Information," *IEEE Transactions on Intelligent Transportation Systems*, vol. 8, no. 3, pp. 379–390, 2007.
- [13] I. Sen and D. Matolak, "Vehicle-to-vehicle channel models for the 5-ghz band," *Intelligent Transportation Systems, IEEE Transactions on*, vol. 9, no. 2, pp. 235–245, 2008.
- [14] M. Torrent-Moreno, D. Jiang, and H. Hartenstein, "Broadcast reception rates and effects of priority access in 802.11-based vehicular ad-hoc networks," *Proceedings of the First ACM workshop on Vehicular ad hoc Networks*, pp. 10–18, 2004.
- [15] M. Torrent-Moreno, S. Corroy, F. Schmidt-Eisenlohr, and H. Hartenstein, "IEEE 802.11-based one-hop broadcast communications: Understanding transmission success and failure under different radio propagation environments," in *Proceedings of the 9th ACM International Symposium on Modeling Analysis and Simulation of Wireless and Mobile Systems*. New York, USA: ACM, 2006, pp. 68–77.
- [16] H. Hartenstein and K. O. Laberteaux, "A tutorial survey on vehicular ad hoc networks," *IEEE Communications Magazine*, vol. 46, no. 6, pp. 164–171, 2008.
- [17] C. Thiemann, M. Treiber, and A. Kesting, "Longitudinal hopping in inter-vehicle communication: Theory and simulations on modeled and empirical trajectory data," *Physical Review E*, vol. 78, p. 036102, 2008.
- [18] W. L. Jin and W. W. Recker, "Instantaneous information propagation in a traffic stream through inter-vehicle communication," *Transportation Research Part B: Methodological*, vol. 40, no. 3, pp. 230–250, 2006.
- [19] W. Jin and W. Recker, "Monte Carlo Simulation Model of Intervehicle Communication," *Transportation Research Record*, vol. 2000, pp. 8–15, 2007.
- [20] X. Wang, "Modeling the process of information relay through inter-vehicle communication," *Transportation Research Part B: Methodological*, vol. 41, no. 6, pp. 684–700, 2007.
- [21] S. Ukkusuri and L. Du, "Geometric connectivity of vehicular ad hoc networks: Analytical characterization," *Transportation Research Part C: Emerging Technologies*, vol. 16, no. 5, pp. 615–634, 2008.
- [22] H. Kim and D. J. Lovell, "Traffic information imputation using a linear model in vehicular ad hoc networks," *IEEE Intelligent Transportation Systems Conference (ITSC '06)*, pp. 1406–1411, 2006.
- [23] D. Choffnes and F. Bustamante, "An integrated mobility and traffic model for vehicular wireless networks," *Proceedings of the 2nd ACM International Workshop on Vehicular ad hoc Networks*, pp. 69–78, 2005.
- [24] M. Fiore, J. Haeri, F. Filali, and C. Bonnet, "Vehicular mobility simulation for VANETs," in *Annual Simulation Symposium*, 2007, vol. 40, p. 301.
- [25] X. Yang and W. Recker, "Simulation studies of information propagation in a self-organizing distributed traffic information system," *Transportation Research Part C: Emerging Technologies*, vol. 13, pp. 370–390, 2005.
- [26] M. Schönhof, M. Treiber, A. Kesting, and D. Helbing, "Autonomous detection and anticipation of jam fronts from messages propagated by intervehicle communication," *Transportation Research Record*, vol. 1999, pp. 3–12, 2007.
- [27] M. Treiber, A. Hennecke, and D. Helbing, "Congested traffic states in empirical observations and microscopic simulations," *Physical Review E*, vol. 62, pp. 1805–1824, 2000.
- [28] A. Kesting, M. Treiber, and D. Helbing, "General lane-changing model MOBIL for car-following models," *Transportation Research Record*, vol. 1999, pp. 86–94, 2007.
- [29] M. Treiber and A. Kesting, "An open-source microscopic traffic simulator," *IEEE Intelligent Transportation Systems Magazine*, vol. 2, no. 3, pp. 6–13, 2010.
- [30] M. Treiber, "<http://www.traffic-simulation.de>."
- [31] A. Chakravarthy, K. Song, and E. Feron, "Preventing automotive pileup crashes in mixed-communication environments," *Intelligent Transportation Systems, IEEE Transactions on*, vol. 10, no. 2, pp. 211–225, 2009.
- [32] D. Helbing, "Traffic and related self-driven many-particle systems," *Reviews of Modern Physics*, vol. 73, pp. 1067–1141, 2001.
- [33] A. Kesting and M. Treiber, "Online traffic state estimation based on floating car data," in *Traffic and Granular Flow '09*. Springer, 2009, in print (preprint <http://arxiv.org/abs/1012.4567>).
- [34] Y. Ma, M. Chowdhury, A. Sadek, and M. Jihani, "Real-time highway traffic condition assessment framework using vehicle-infrastructure integration (vii) with artificial intelligence (ai)," *IEEE Transactions on Intelligent Transportation Systems*, vol. 10, no. 4, pp. 615–627, 2009.